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ON THE EXPRESSION FOR THE HESSIAN OF A BINARY QUANTIC IN TERMS OF THE ROOTS.

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In Salmon's Higher Algebra (p. 185) is given an expression for the Hessian of a binary cubic in terms of the roots, viz:—

$$-18H = a_0^2 \Sigma (\alpha - \beta)^2 (x - \gamma y)^2,$$

which is derived by a special method from the harmonic relations between the roots of the cubic and of the Hessian. On page 192 is given, without proof, for the Hessian of a quartic the expression

$$-48H = a_0^2 \Sigma (\alpha - \beta)^2 (x - \gamma y)^2 (x - \delta y)^2;$$

and on page 259 for the Hessian of a quintic the expression

$$-100H = a_0^2 \Sigma (\alpha - \beta)^2 (x - \gamma y)^2 (x - \delta y)^2 (x - \varepsilon y)^2.$$

I propose to prove that this form is general, and that the expression for the Hessian of the binary n-ic, in terms of the roots, is

$$-n^2(n-1)H = a_0^2 \Sigma (\alpha_1 - \alpha_2)^2 (x - \alpha_3 y)^2 (x - \alpha_4 y)^2 \ldots (x - \alpha_n y)^2;$$

for this expression is evidently from its form a covariant (p. 125), and of the degree $2(n-2)$ in x, y, and of the order 2 in the coefficients of the quantic (p. 59); again, the covariant $\frac{d^2u}{dx^2} \cdot \frac{d^2u}{dy^2} - \left(\frac{d^2u}{dxdy}\right)^2$, which is the Hessian of the quantic u (with a numerical factor), is evidently of the same degree and order as the above covariant; hence, to show that these two covariants differ only by a factor (involving n alone), we must show that there can be but *one* covariant of the degree $2(n-2)$ in x, y, and of the order 2 in the coefficients. Now, it is seen from page 134 that the source of either of these covariants has the weight $\frac{1}{2}(n\theta - p)$, which $= 2$, when $\theta = 2$ and $p = 2(n-2)$; hence the source of either covariant is of the form $a_0 a_2 + \lambda a_1^2$, and, in order that this may be reduced to zero by the operator $a_0 \frac{d}{da_1} + 2a_1 \frac{d}{da_2} + \ldots$, we must have $\lambda = -1$, therefore there is but one source, and but one covariant of this degree and order.

To determine the multiplier, take the special binary n-ic

$$a_0x^n + na_1x^{n-1}y,$$

wherein one root is $-\frac{na_1}{a_0}$ and the other $n-1$ roots are zero, and compare the coefficients of $x^{2(n-2)}$ in the two expressions for the covariant: the coefficient of $x^{2(n-2)}$ in the Hessian is $a_0a_2 - a_1^2$, which in this case becomes $-a_1^2$; and the coefficient of $x^{2(n-2)}$ in

$$\Sigma(\alpha_1-\alpha_2)^2(x-\alpha_3y)^2(x-\alpha_4y)^2\dots(x-\alpha_ny)^2$$

is $\Sigma(\alpha_1-\alpha_2)^2$, which becomes $(n-1)\left(\frac{na_1}{a_0}\right)^2$; hence the multiplier is determined, and we have the identity

$$-n^2(n-1)H = a_0^2\Sigma(\alpha_1-\alpha_2)^2(x-\alpha_3y)^2(x-\alpha_4y)^2\dots(x-\alpha_ny)^2.$$